Exercise 47

Researchers measured the average blood alcohol concentration $C(t)$ of eight men starting one hour after consumption of 30 mL of ethanol (corresponding to two alcoholic drinks).

t (hours)	1.0	1.5	2.0	2.5	3.0
$C(t)$ (mg/mL)	0.33	0.24	0.18	0.12	0.07

(a) Find the average rate of change of C with respect to t over each time interval:

(i) $[1.0, 2.0]$ (ii) $[1.5, 2.0]$

(iii) $[2.0, 2.5]$ (iv) $[2.0, 3.0]$

In each case, include the units.

(b) Estimate the instantaneous rate of change at $t = 2$ and interpret your result. What are the units?

Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

Solution

Calculate the average rate of change of C with respect to t over each of the time intervals.

(i) $[1.0, 2.0]$ $$\frac{C(2.0) - C(1.0)}{2.0 - 1.0} = \frac{0.18 - 0.33}{1.0} = -0.15 \ \frac{\text{mg}}{\text{mL} \cdot \text{hour}}$$

(ii) $[1.5, 2.0]$ $$\frac{C(2.0) - C(1.5)}{2.0 - 1.5} = \frac{0.18 - 0.24}{0.5} = -0.12 \ \frac{\text{mg}}{\text{mL} \cdot \text{hour}}$$

(iii) $[2.0, 2.5]$ $$\frac{C(2.5) - C(2.0)}{2.5 - 2.0} = \frac{0.12 - 0.18}{0.5} = -0.12 \ \frac{\text{mg}}{\text{mL} \cdot \text{hour}}$$

(iv) $[2.0, 3.0]$ $$\frac{C(3.0) - C(2.0)}{3.0 - 2.0} = \frac{0.07 - 0.18}{1.0} = -0.11 \ \frac{\text{mg}}{\text{mL} \cdot \text{hour}}$$

For the best estimate of the instantaneous rate of change at $t = 2$, take the average of the average rates taken over $[1.5, 2.0]$ and $[2.0, 2.5]$, the smallest time intervals about $t = 2$.

$$\frac{(-0.12) + (-0.12)}{2} = -0.12 \ \frac{\text{mg}}{\text{mL} \cdot \text{hour}}$$

This indicates that two hours after having two alcoholic drinks, the blood alcohol concentration is decreasing at a rate of 0.12 mg/mL per hour.